A Leader-Follower Approach to Formation Control of Multiple Non-Holonomic Mobile Robots

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Abstract - Many cooperative tasks in real world environments need the robots to maintain some desired formations when moving. Formation control refers to the problem of controlling the relative position and orientation of robots in a group, while allowing the group to move as a whole. In this paper, a novel leader-follower formation control law for multiple non-holonomic mobile robots based on kinematic models and trajectory tracking techniques is proposed.

I. INTRODUCTION

Research interest in formation and cooperative control of robotic systems has recently grown enormously in the control community. The usage of groups of robots has several advantages compared to a single robot. For instance, the tasks that can be accomplished are inherently more complex than those that can be accomplished by a single robot. Also, the system becomes more flexible (since robots can have a variety of roles, the same group of robots can be employed for many different objectives) and robustness (system can be designed so that a robot can take over the tasks of another robot in case of failure). The range of applications includes: (i) dangerous tasks on locations where it is harmful for humans to work, e.g. areas of toxic contamination, nuclear contamination or forest fires; (ii) tasks that cannot be performed by a single robot, e.g. transporting or repositioning of large objects; (iii) tasks that scale up or down in time, e.g. certain tasks may require a decreasing or increasing amount of robots as the operation proceeds over time; (iv) exploration, searching and rescue tasks.

In the literature, three different control approaches for mobile robot formation are described: behaviour-based approach [1-6], virtual-structure approach [7-8] and leader–follower approach [9-16].

In the behaviour-based approach, the selection of robots behaviour (e.g. obstacle avoidance, target seeking) is based on some rules and each behaviour has its specific purpose or task. The advantage of this approach is that it is natural to derive control strategies when robots have multiple competing objectives and that the robots are controlled in a decentralized mode. Therefore, it is suitable for a large number of robots. A disadvantage is complex mathematical analysis and it is hard to guarantee exact formation control and stability of the system as a whole.

In the leader-follower approach, one of the robots is assigned as the leader and others are followers. The leader follows a predefined trajectory, while the followers are keeping the position and direction with a certain distance to the leader. The advantage of this approach is that it is easy to understand and implement. However, this approach has some disadvantages. It asks for a centralized control strategy (followers use the position of the leader robot as a control input), which makes it less suitable for a large number of robots. There is no explicit feedback from the followers to the leaders. For example, if the leader moves too fast, or the follower is blocked by an obstacle, the formation will be destroyed. Another disadvantage is that, if the leader has failed, the entire formation can not be kept and the consequence will be very serious. One method which is often used to maintain the formation is to define an internal shape variable of the follower robot according to the relative angle and relative distance from its leader or defining the relative distance from two of its leaders. However, the leader-follower approach will not maintain its formation if the followers are perturbed, which results in a limited robustness with respect to the formation.

In the virtual-structure approach the robot formation is considered as a virtual rigid structure (e.g. a circle, square, etc). The advantage of this approach is that the formation is easy to describe. The main difference between the leader-follower approach and the virtual structure approach is that for the virtual structure approach extra parameters that specify the formation, so called mutual parameters, are added to the controllers. If these parameters are added to the control feedback, the structure is more robust, even if some robots are perturbed. A disadvantage is that it is often more difficult to determine stability than for the leader-follower approach.

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In this paper, we investigate leader-following problem by assigning a virtual leader robot for each robot to guarantee the formation stability. Virtual formation constitutes of virtual leader and virtual followers. The positions of virtual followers are defined as offsets from the origin of the virtual leader coordinates in the directions of robot local coordinate system.

The rest of the paper is organized as follows. First, we describe the model of unicycle robot in section 2. Then, the main results with design procedures and stability analysis are presented in section 3. In section 4, simulation results are given. Finally, the paper ends with concluding remarks in section 5.

II. ROBOT MODEL

In this paper, we deal with a group of unicycle robots. Mathematically the kinematic model of a unicycle robot is described by the following equations:

$$\dot{x} = v\cos\theta, \quad \dot{y} = v\sin\theta, \quad \dot{\theta} = \omega, \tag{1}$$

where v, ω denote the translational and rotational velocities which are assumed to be the control input of the system. Mobile robots of this type are subject to non-holonomic constraints [18]:

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0. \tag{2}$$

It is important to mention that dynamics of a robot is not modelled by the kinematic model. Therefore, it is assumed that robot exactly realizes velocity commands v and ω. Of course, due to its body and actuator dynamics, and also non-idealities such as friction, gear backslash, wheel slippage, actuator dead zone and saturation, a robot cannot exactly realize velocity commands. Nevertheless, as we have assumed that the command trajectory is feasible for the robot, robot can approximately track the reference velocity commands.

In simulations we use the robot model for robot soccer [19]. The robot model is developed in MATLAB® Simulink and simulations are done in MATLAB® Simulink also.

Let $v_{\max}$ and $\omega_{\max}$ are the absolute maximum linear and angular velocities of the robot, respectively. As a safety measure, in case when the formation tracking controller temporarily generates command velocities higher than robot limitations, velocity saturation will occur. In order to preserve the curvature radius originated from v and ω, a velocity scaling is needed as follows. If the scaled down values of linear and angular velocities are v_s and ω_s respectively, we have [17]:

$$\begin{array}{c} \delta = \max\{|v|/v_{\max}, |\omega|/\omega_{\max}, 1\} \\ v_s = sign(v)v_{\max}, \omega_s = \omega/\delta \quad if \quad \delta == |v|/v_{\max} \\ v_s = v/\delta, \omega_s = sign(\omega)\omega_{\max} \quad if \quad \delta == |\omega|/\omega_{\max} \\ v_s = v, \omega_s = \omega \quad if \quad \delta == 1 \end{array}. \tag{3}$$

III. VIRTUAL ROBOT TRACKING BASED FORMATION CONTROLLER

In the leader-follower based formation control, the desired poses of followers can be thought of as offsets to the origin of the leader robot coordinate system at any given time. These offsets mimic virtual robots which the actual followers must track. Note that these offsets may be time invariant or time variant. In this paper, for simplicity, we assume that offset is time invariant. Tracking paths and velocities of the virtual robot require a combination of a nominal feed-forward command, which calculates the pose of the virtual robot and it's linear and angular velocities to which the actual designated follower must reach to, with a feedback action on the error.

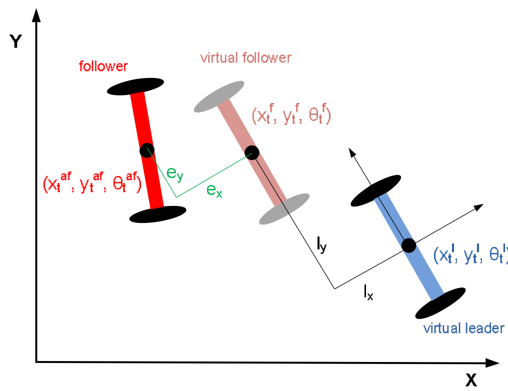

Figure 1. Leader-follower based formation control

Assuming that the virtual leader robot's pose at time t is $(x_t^l, y_t^l, \theta_t^l)$ and linear and angular velocities are (v_t^l, ω_t^l). The desired position for a virtual follower (x_t^f, y_t^f) can be described as offsets l_x and l_y from the origin of the virtual leader robot coordinates in the directions of robot X and robot Y coordinates, respectively:

$$\begin{bmatrix} x_t^f \\ y_t^f \end{bmatrix} = \begin{bmatrix} \cos\theta_t^l & -\sin\theta_t^l \\ \sin\theta_t^l & \cos\theta_t^l \end{bmatrix} \begin{bmatrix} l_x \\ l_y \end{bmatrix} + \begin{bmatrix} x_t^l \\ y_t^l \end{bmatrix}. \tag{4}$$

The derivatives of desired virtual follower pose variables are:

$$\begin{bmatrix} \dot{x}_t^f \\ \dot{y}_t^f \\ \dot{\theta}_t^f \end{bmatrix} = \begin{bmatrix} \cos\theta_t^l & -l_x\sin\theta_t^l - l_y\cos\theta_t^l \\ \sin\theta_t^l & l_x\sin\theta_t^l - l_y\cos\theta_t^l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_t^l \\ \omega_t^l \end{bmatrix}. \tag{5}$$

The desired linear and angular velocities of virtual follower are given by:

$$v_t^f = \pm\sqrt{(\dot{x}_t^f)^2 + (\dot{y}_t^f)^2}, \tag{6}$$

$$\omega_t^f = \frac{\ddot{y}_t^f \dot{x}_t^f - \ddot{x}_t^f \dot{y}_t^f}{(\dot{x}_t^f)^2 + (\dot{y}_t^f)^2}. \tag{7}$$

The ω_t^f is derived through defining θ_t^f as:

$$\theta_t^f = a\tan 2(\dot{y}_t^f, \dot{x}_t^f) + k\pi, \quad k = 0,1, \tag{8}$$

where $k=0$ is for forward motion and $k=1$ is for backward motion. Note if l_x and l_y are constants and $(\dot{v}_t^l, \dot{\omega}_t^l) = (0,0)$, than angular velocities of the virtual leader and the virtual follower have the same value $\omega_t^f = \omega_t^l$.

The problem of minimizing difference between virtual robot, i.e. reference configuration, and actual designated follower robot configuration is equivalent to the problem of stabilization of tracking error dynamics. The tracking error $e = [e_x, e_y, e_\theta]^T$ can be defined in the configuration space of the robot, however it is convenient to transform it to the robot local coordinate system. This transformation is performed as follows

$$\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta_t^{af} & \sin\theta_t^{af} & 0 \\ -\sin\theta_t^{af} & \cos\theta_t^{af} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_t^f - x_t^{af} \\ y_t^f - y_t^{af} \\ \theta_t^f - \theta_t^{af} \end{bmatrix}, \quad (9)$$

where error component e_x is the longitudinal error, e_y is the orthogonal error, while e_θ is the orientation error as illustrated in Figure 1 and $(x_t^{af}, y_t^{af}, \theta_t^{af})$ is the actual follower pose. By taking a derivative of the above equation and taking into account robot kinematics (1) and (2), the error dynamics become [20]

$$\begin{aligned} \dot{e}_x &= \omega_t^{af} e_y - v_t^{af} + v_t^f \cos e_\theta \\ \dot{e}_y &= -\omega_t^{af} e_x + v_t^f \sin e_\theta \\ \dot{e}_\theta &= \omega_t^f - \omega_t^{af} \end{aligned} . \quad (10)$$

To make the formation error (10) converge to zero, we first stabilize e_x with v_t^{af}. Because e_y cannot be directly controlled, we choose appropriate ω_t^{af} to decrease the effect of $\omega_t^{af} e_y$ and $\omega_t^{af} e_x$.

If we choose

$$v_t^{af} = k_x e_x + v_t^f \cos e_\theta , \quad (11)$$

the close-loop dynamics of $\dot{e}_x$ become

$$\dot{e}_x = -k_x e_x + \omega_t^{af} e_y , \quad (12)$$

where k_x is a positive constant. From (12) we can conclude that, if e_y is zero, then e_x can exponentially converge to zero.

To determine the control input ω_t^{af}, we chose a Lyaponuv function as

$$V(t) = \frac{e_x^2}{2} + \frac{e_y^2}{2} + \frac{1-\cos e_\theta}{k_y} , \quad (13)$$

with k_y as a positive constant.

The time derivative of Lyaponuv function (13) with (10) and (11) is

$$\dot{V}(t) = -k_x e_x^2 + \sin e_\theta \left[v_t^f e_y + \frac{\omega_t^f - \omega_t^{af}}{k_y} \right]. \quad (14)$$

To make the derivative of the Lyapunov function (14) negative, we choose

$$\omega_t^{af} = \omega_t^f + v_t^f k_y e_y + k_\theta k_y \sin e_\theta . \quad (15)$$

where k_θ is a positive constant.

Finally, from (14) and (15) we obtain

$$\dot{V}(t) = -k_x e_x^2 - k_\theta \sin^2 e_\theta \le 0 . \quad (16)$$

Now we analyse the stability of the formation tracking control law.

Theorem 1: Under the control law (11) and (15), the equilibrium of system (9) is locally stable.

Proof: $V(t)$ is positive definite and $\dot{V}(t)$ with (11) and (15) is negative semi-definite around the origin. Therefore, $V(t)$ is a Lyaponuv function, which indicates the local stability [21]

Remark 1: In [20], the local stability is solved under condition $v_t^f > 0$. Note that in our case (16) does not depend on v_t^f.

Theorem 2: Assume that (i) v_t^f and ω_t^f are continuous, (ii) v_t^f, ω_t^f, k_x, k_y, k_θ are bounded, (iii) k_x, k_y, k_θ are positive constants and (iv) $\dot{v}_t^f$ and $\dot{\omega}_t^f$ are sufficiently small. Under these condition, $e=0$ is uniformly asymptotically stable over $[0,\infty)$.

Proof: We first linearize the nonlinear system (10) around the origin:

$$\dot{e} = Ae = \begin{bmatrix} k_x & \omega_t^f & 0 \\ -\omega_t^f & 0 & v_t^f \\ 0 & -k_y v_t^f & -k_\theta k_y \end{bmatrix} e . \quad (17)$$

The characteristic equation for A is:

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 , \quad (18)$$

where

$$\begin{aligned} a_0 &= 1 \\ a_1 &= k_\theta k_y + k_x \\ a_2 &= (\omega_t^f)^2 + k_y (v_t^f)^2 + k_x k_y k_\theta \\ a_3 &= k_x k_y (v_t^f)^2 + k_\theta k_y (\omega_t^f)^2 \end{aligned} . \quad (19)$$

Since all coefficients of (18) are positive and $a_1 a_2 > a_0 a_3$, the real parts of all roots are negative according the Routh criterion [22]. Therefore, the theorem was proved.

Let the actual pose of leader robot be $(x_t^{al}, y_t^{al}, \theta_t^{al})$ and its velocities $(v_t^{al}, \omega_t^{al})$. The leader robot tracks virtual leader robot with zero offsets $(l_x, l_y) = (0,0)$. The design of control law for the leader is the same as for the follower robot. In equations (9)-(19) parameters $x_t^f, y_t^f, \theta_t^f, x_t^{af}, y_t^{af}, \theta_t^{af}, v_t^{af}, \omega_t^{af}$ should be replaced with $x_t^l, y_t^l, \theta_t^l, x_t^{al}, y_t^{al}, \theta_t^{al}, v_t^{al}, \omega_t^{al}$.

IV. SIMULATION RESULTS

For simulation purposes an 8-shaped trajectory was used as is illustrated in Figure 2. The group consists of three robots, one leader robot and two followers. Follower 1 is in parallel formation with the leader while follower 2 is in serial formation with the leader. Since we describe formation in Cartesian coordinates with respect to the leader robot (4), we will show in simulations that our control law is capable of dealing with two mentioned formation. In [10] formation is described in polar coordinates with respect to the leader robot and it is proposed two control laws, one for parallel formation and one for serial formation.

In simulation, we choose following values of constants for our control law: $k_x = 1$, $k_y = 1$ and $k_\theta = 10$. Leader robot and virtual leader robot initial poses were $[x_t(0), y_t(0), \theta_t(0)] = [0,\ 0,\ \pi/2]$. Initial poses of follower 1 and 2 were set to the values $[x_t^f(0), y_t^f(0), \theta_t^f(0)] = [-0.1,\ -0.05,\ \pi/2]$ and $[x_t^f(0), y_t^f(0), \theta_t^f(0)] = [0,\ 0.1,\ \pi/3]$, respectively. Desired formation for the follower 1 was $(l_x, l_y) = (0,\ 0.05)$ while for the follower 2 was $(l_x, l_y) = (-0.02,\ -0.05)$. Initial velocities for all robots were zero. Reference linear and angular velocities of virtual leader for left and right part of 8-shaped trajectory were set to the values $(v_t^l, \omega_t^l) = (0.1,\ 0.4)$ and $(v_t^l, \omega_t^l) = (0.1,\ -0.4)$, respectively. Note that linear velocities of virtual followers v_t^f (see Figure 6 and 8) depend on desired formation, i.e. offset.

Figure 3 presents how the leader robot tracks the virtual robot. Since all initial tracking errors were set to zero, leader robot quickly reaches reference values of linear and angular velocities (Figure 4). Initial position error of follower 1 which can be seen on Figure 5, causes slow reaching of reference velocities value (Figure 6). An initial pose (position and orientation) error of follower 2 causes a big value of robots angular velocity on the beginning of fetching desired formation (Figure 8).

V. CONCLUSION

This paper proposes a stable leader-follower formation control law based on kinematic model and trajectory tracking technique. Offset that define formation can be time invariant or time variant. The proposed control law may be used for pure trajectory tracking (with and without initial tracking error). Stability of the control law is proved through the use of a Lyaponuv function. Simulation results confirm theoretical results.

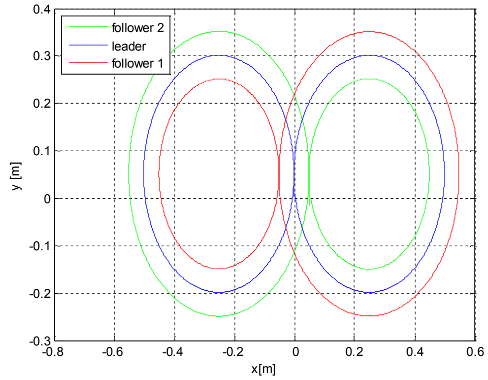

Figure 2. The paths of virtual leader and virtual followers

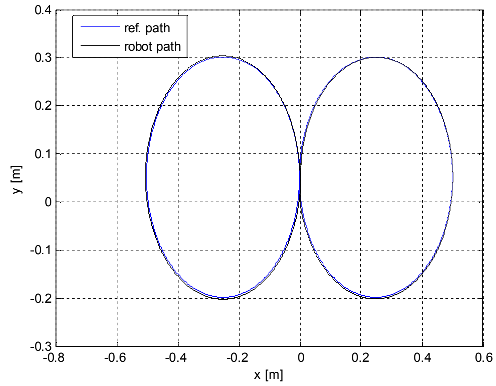

Figure 3. Formation control of the leader robot

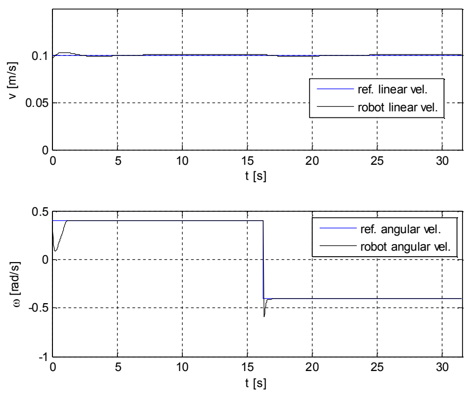

Figure 4. Linear and angular velocities of the leader robot

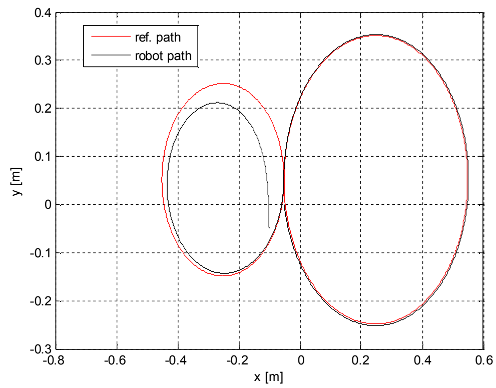

Figure 5. Formation control of the follower 1

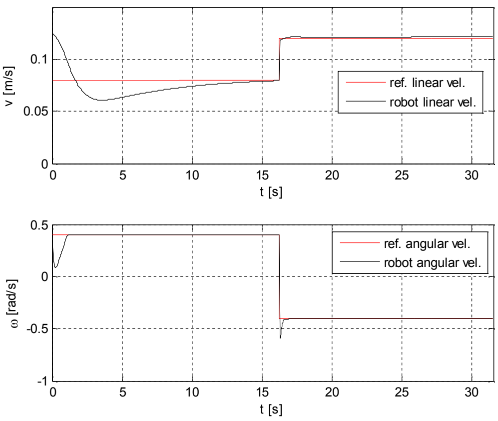

Figure 6. Linear and angular velocities of the follower 1

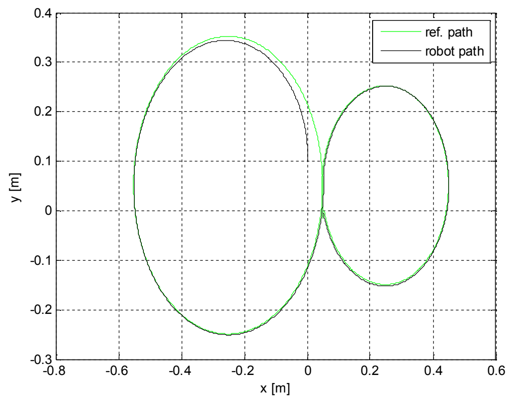

Figure 7. Formation control of the follower 2

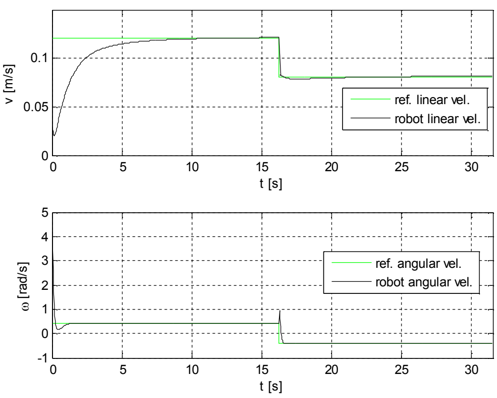

Figure 8. Linear and angular velocities of the follower 2

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