Longitudinal Spacing Control of Vehicles in a Platoon for Stable and Increased Traffic Flow*

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Abstract— **Vehicle platoon systems are a promising approach for new transportation systems because of their innovative capabilities. A basic problem in platoon systems is the control of the inter-vehicle spacing. In this paper, a new nonlinear longitudinal spacing control of vehicles in a platoon is proposed, which takes into account both vehicle characteristics and road conditions. It ensures the string stability of a string of vehicles, traffic flow stability and in comparison with the common constant time-gap spacing control policy increases the capacity of the traffic flow.**

I. INTRODUCTION

The idea of longitudinal vehicle control has grown very quickly and became very attractive with the increasing problems of traffic congestion and safety. Longitudinal vehicle control system controls the longitudinal motion of the vehicle, such as its longitudinal velocity, acceleration or longitudinal distance from the preceding vehicle, i.e. the platoon leader, in the same lane, by using throttle and brake controllers, all this with a primary goal of reducing the efforts of the driver.

The cruise control (CC) system [1] works in a way that the driver brings the vehicle speed to the wanted level and then by pressing the right button activates the system that maintains the wanted speed of the vehicle by controlling the electronic throttle of the engine. In case a driver sees a slower vehicle in the front he should react by pressing the brake-shoe and the control of the vehicle will be given back to him. A more modern version of this system – adaptive cruise control (ACC), involves also the controlling of the brake system. In case of a slower vehicle in the front, which is detected with an on-board distance radar sensor, the vehicle automatically slows down and switches from the maintaining constant speed mode to the maintaining constant distance mode, i.e. to the longitudinal spacing control, with respect to the slower preceding vehicle. If the preceding vehicle changes the traffic lane, the ACC system accelerates the vehicle to the speed that was previously given to it.

With the rapid evolution of vehicles and due to the emerging problems of traffic congestion in overpopulated cities, the program PATH (Partners for Advanced Transit and Highways) proposed the concept of controlling vehicle platoons where the vehicles travel together with a close spacing. It was estimated that the traffic capacity is about four times the capacity of a typical highway if all the vehicles travel in the closely packed platoons [2]. Traveling with a close spacing between preceding and following vehicle may be extremely dangerous. One of the major mistakes that drivers do in traffic is following the preceding vehicle with a very small distance from it. It is estimated that 15% of car crashes refers to the collision of vehicles in a platoon [3]. A bigger distance between vehicles gives more time to react when the vehicle in the front suddenly brakes. This way a driver gets more space to "escape" from a dangerous situation. For these reasons it is necessary that the ACC system, depending on the environment conditions (weather condition, road characteristics), and vehicle characteristics (first of all brakes quality), ensures a safe distance to the preceding vehicle.

Inspired by the concept of platoon [2, 4, 5], our principal objective is to design a longitudinal spacing control system which can enhance vehicle safety and increase traffic capacity. More specifically, the proposed longitudinal spacing controller possesses the following properties: (i) it guarantees string stability in a platoon of vehicles; (ii) it leads to increased traffic capacity and stable traffic flow; (iii) it explicitly takes into account the vehicle's braking capability and road conditions; (iv) it could be used for a wide range of vehicle's velocity.

The rest of the paper is structured as follows. The architecture of the longitudinal spacing control system is described in chapter two. The definitions of string stability of a platoon of vehicles and traffic flow stability are given in chapter three. In chapter four, it is analyzed the constant time-gap spacing control policy and the proposed spacing control policy. The influence of the spacing control policy on the traffic flow is analyzed in chapter five. Chapter six is the conclusion of this paper.

II. ARCHITECTURE OF LONGITUDINAL SPACING CONTROL SYSTEM

The longitudinal spacing control system is designed to be hierarchical [6], with an upper level controller and a lower level controller as shown in Fig. 1. The preceding vehicle is independent of following vehicle, travels on its own. It is assumed that there is no any communication between the two

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vehicles. The longitudinal motion of the preceding vehicle and following vehicle are measured by the upper-level controller in order to compute the desired acceleration commands. Since the desired acceleration is not a true control input, a lower-level controller is required to determine either a throttle (regulates the flow of the air in the burning chamber) or brake input in order to track the desired acceleration computed by the upper-level controlled. The relation between the openness of the throttle and the developed torque on the engine axle is a nonlinear function and it is usually given in a table or in a diagram. Brake algorithm takes the desired acceleration and subtracts it to the actual vehicle acceleration and provides a brake torque. The information of the longitudinal motion of controlled vehicle is fed back to the upper-level controller to establish a feedback closed loop system.

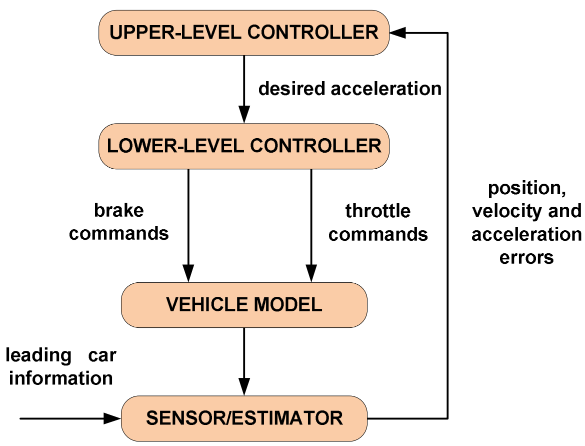

Figure 1. Two-level structure for longitudinal spacing control system

III. STABILITY DEFINITIONS

A. *Individual vehicle stability*

The vehicle following control law is said to provide individual vehicle stability if the spacing error of the following vehicle converges to zero when the preceding vehicle is operating at constant speed. Spacing error in this definition refers to the difference between the actual and desired spacing from the preceding vehicle. For the *i*-th vehicle (see Fig. 2) the spacing error is defined as

$$\delta_i = x_i - x_{i-1} + S_i \,, \tag{1}$$

where x_i is location of the *i*-th vehicle and S_i is desired inter-vehicle spacing of the *i*-th vehicle and it includes the preceding vehicle length l_{i-1}.

The *i*-th vehicle is said to provide individual vehicle stability if the following condition is satisfied:

$$\ddot{x}_i \to 0 \quad \Rightarrow \delta_i \to 0 \,. \tag{2}$$

B. *String stability of a string of vehicles*

The string stability of a platoon of vehicles refers to a property in which spacing errors are guaranteed not to amplify as they propagate towards the tail of the string. This property ensures that any spacing error present at the head of the platoon does not amplify into a large error at the tail of the platoon. Equivalently, this property ensures that the variation of preceding vehicle speed will not result in amplified fluctuations in the following vehicle speed. The spacing error is expected to be non-zero during acceleration and deceleration of the preceding vehicle.

The oscillating behavior of the preceding vehicle might become so severe at some point in the string that the vehicles reach their accelerating or braking limits. Hence, a string unstable platoon may result a "harmonica effect" which may result in traffic jams or even collisions. String stability must not be confused with "ordinary stability", i.e. stability of solutions of differential equations and of trajectories of dynamical systems under small perturbations of initial conditions, as they evolve in time. String stability considers the propagation of disturbances from vehicle to vehicle, i.e. as they evolve in vehicle index.

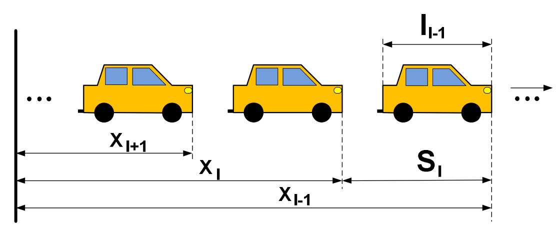

Figure 2. Vehicle platoon

Sheikholeslam and Desoer [7] were the first who set up the condition for string stability. The more formalized and generalized definition of string stability and a sufficient condition for string stability was given by Swaroop [8, 9].

Consider vehicle following system, such as a vehicle platoon as shown in Fig. 2. The spacing error for the *i*-th vehicle is defined in (1). A sufficient condition for string stability is that [8, 9]:

$$\|\delta_i\|_\infty \le \|\delta_{i-1}\|_\infty \,. \tag{3}$$

Let $H(s)$ be the transfer function relating the spacing errors of consecutive vehicles

$$H(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} \,, \tag{4}$$

where $\delta_i(s)$ is Laplace transform of $\delta_i(t)$. Let $h(t)$ be the impulse response of $H(s)$, thus the condition (3) for string stability becomes

$$\|h(t)\|_1 \le 1 \,. \tag{5}$$

The condition (5) can be replaced by [9]:

$$\|H(s)\|_\infty \le 1 \,, \tag{6}$$

$$h(t) > 0 \,. \tag{7}$$

The condition $\|H(s)\|_\infty \le 1$ ensures that $\|\delta_i\|_2 \le \|\delta_{i-1}\|_2$ which means that the energy in the spacing error signal decreases as the spacing error propagates towards the tail of the platoon. The condition $h(t) > 0$ ensures that the steady state spacing errors of the vehicles in the platoon have the same sign (positive/negative spacing error implies that a vehicle is closer/further than desired).

In chapter four, the conditions (6) and (7) will be used to evaluate the spacing policy for the vehicle longitudinal control. Moreover, it is much easier to design a system to ensure (6) and (7) are satisfied than to design a system to ensure (5) is satisfied.

C. *Traffic flow stability*

The conception of the traffic flow stability differs from the conception of string stability. While string stability does not consider entering/leaving a vehicle in/from the platoon, the traffic flow stability does. The difference between string stability of a platoon of vehicles and traffic flow stability was recognized for the first time by Swaroop [10].

The concept of the traffic flow stability is illustrated in Fig. 3. The perturbation of the nominal traffic flow density of the main traffic lane is happening at the exit of the secondary traffic lane due to the traffic merging from secondary into the main traffic lane. An unstable traffic flow means that such a density perturbation feels on every point upstream from the source without attenuation. At a stable traffic flow the perturbation attenuates upstream.

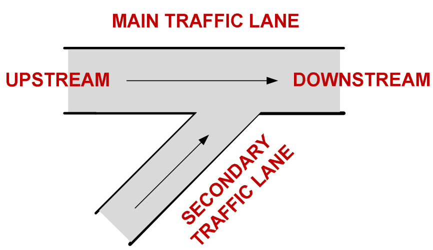

Figure 3. Traffic flow illustration

It is obvious that the dynamics of the traffic greatly depends on the properties of the longitudinal spacing control system. Therefore, when designing the longitudinal spacing control system it is necessary to examine its influence on the traffic dynamics. For the sake of simplicity, we assume a one-lane road (overtaking is impossible) with all vehicles having equal spacing controllers.

The fundamental traffic flow diagram is shown in Fig. 4. We divide the various traffic states into traffic regimes according to the slope of the characteristics. When traffic density lies below the critical density, we speak of *free flow* – slope of the characteristics is positive. During this regime vehicles are not impeded by other traffic and they travel at maximum speed. This speed is dependent, amongst other things, on the design speed of a road, the speed restrictions in operation at any particular time and the weather. When traffic density lies between the capacity density and the maximum density we speak of *congested flow* - slope of the characteristics is negative. It is the regime in which tailbacks develop.

The traffic flow is modeled as a continuum. The most elementary continuum traffic flow model was the first order model developed by Lighthill & Whitham [11], based around the assumption that the number of vehicles is conserved between any two points if there are no entrances (sources) or exits (sinks). Their results are based on the theory of kinematic waves.

In order to analyze the influence of the spacing control policy on the traffic flow dynamics, consider the spacing policy S is a function of the vehicle velocity v,

$$S = g(v)\,. \tag{8}$$

This relation can be translated as the following equation:

$$\frac{1}{\rho} = g(v) \quad \Rightarrow \quad v = z(\rho) := g^{-1}\left(\frac{1}{\rho}\right). \tag{9}$$

Analogous to the flow of a fluid, traffic flow Q at any point is defined as

$$Q = \rho v = \rho z(\rho)\,. \tag{10}$$

The evolution of traffic density is determined by the conservation of mass equation (it states that if, at a specific location, the flow coming in from the left is less than the flow going out to the right ($\partial_x Q > 0$), then density has to decrease at that location ($\partial_t \rho < 0$)):

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0 \underset{Q\ depends\ on\ \rho\ only}{\Rightarrow} \frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial \rho}\frac{\partial \rho}{\partial x} = 0\,. \tag{11}$$

Substituting for Q from (10), we get

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho z(\rho))}{\partial \rho}\frac{\partial \rho}{\partial x} = 0\,. \tag{12}$$

Let ρ_0 be a base solution for the density. In order to study the stability of the base solution, consider small density perturbations, $\varepsilon\rho_p$ to the base solution ($\varepsilon\rho_p << \rho_0$). Substituting $\rho(x,t) = \rho_0 + \varepsilon\rho_p(x,t)$ into (12) and neglecting second-order terms in ε, we come to the evolution of a density perturbation of the traffic flow

$$\frac{\partial \rho_p}{\partial t} + c\frac{\partial \rho_p}{\partial x} = 0\,, \tag{13}$$

were the traveling speed of the small density perturbation (kinematic wave speed) is:

$$c = \left.\frac{\partial Q}{\partial \rho}\right|_{\rho_0} = z(\rho_0) + \rho_0 \frac{\partial z}{\partial \rho}(\rho_0)\,. \tag{14}$$

c is the tangent slope of the fundamental flow diagram (see Fig. 4) at ρ_0. The solution of (13) is traveling wave $\rho_p = A(x - ct)$. If $c > 0$, the solution is forward traveling wave. If $c < 0$, the solution is backward traveling wave. In the traffic flow dynamics, when $c < 0$, small density perturbations are propagated upstream without any attenuation and the traffic flow is unstable. In contrast, the traffic flow stability can be ensured when $c > 0$.

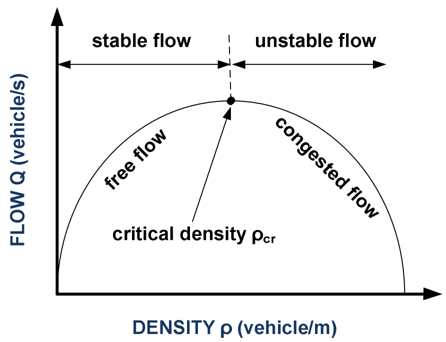

Figure 4. Fundamental flow diagram

IV. LONGITUDINAL SPACING POLICIES

A. *Constant time-gaps spacing policy*

The most common longitudinal spacing policy used by researchers is the constant time-gap (CTG) spacing policy [4, 6, 9, 12, 14]. In the CTG spacing policy, the desired inter-vehicle spacing for the *i*-th vehicle varies linearly with velocity:

$$S_i = L + t_g v_i \,, \tag{15}$$

where L is a constant that includes the vehicle length l_{i-1} of the preceding vehicle (see Fig. 2), t_g is the time-gap between preceding and following vehicle and v_i is the velocity of the *i*-th vehicle.

The spacing error for the *i*-th vehicle under CTG spacing policy is given by:

$$\delta_i = x_i - x_{i-1} + L + t_g v_i = \varepsilon_i + L + t_g v_i \,. \tag{16}$$

The control law developed in [12]

$$\dot{v}_{i\_des} = -\frac{\dot{\varepsilon}_i + \lambda\delta_i}{t_g} \,, \tag{17}$$

ensures that the spacing error (16) converges to zero. The dynamic of δ_i is set as $\dot{\delta}_i = -\lambda\delta_i$, where λ is a positive control gain.

In the presence of the lower controller and actuator dynamics (see Fig. 1), the desired acceleration of *i*-th vehicle a_{i_des} is not obtained instantaneously but instead satisfies the dynamics approximated by equation [6]:

$$\tau \dot{a}_i + a_i = a_{i\_des} \,, \tag{18}$$

where a_i is the actual acceleration of *i*-th vehicle and τ is time lag describing vehicle dynamics.

Substituting a_{i_des} from (17) into (18), we get

$$\tau\dddot{x}_i + \ddot{x}_i = -\frac{\dot{\varepsilon}_i + \lambda\delta_i}{t_g} \,. \tag{19}$$

Differentiating δ_i twice from (16) and substituting $\dddot{x}_i$ from (19), we get

$$\ddot{\varepsilon}_i = \ddot{\delta}_i + \frac{1}{\tau}(\dot{\delta}_i + \lambda\delta_i) \,. \tag{20}$$

Consider (16), the difference between errors of successive vehicles can be written as

$$\delta_i - \delta_{i-1} = \varepsilon_i - \varepsilon_{i-1} + t_g \dot{\varepsilon}_i \,. \tag{21}$$

Using (20) to substitute in (21), a dynamic relation between δ_i and δ_{i-1} can be obtained. Transfer function relating the spacing errors of consecutive vehicles is:

$$H(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{s + \lambda}{t_g \tau s^3 + t_g s^2 + (1 + \lambda t_g)s + \lambda} \,. \tag{22}$$

By using the above transfer function, the string stability of the CTG spacing policy can be analyzed. In [9] it was shown that the condition $\|H(s)\|_\infty \le 1$ is ensured only if:

$$t_g \ge 2\tau \,. \tag{23}$$

The requirement $\|H(s)\|_\infty \le 1$ can always be satisfied if we choose a sufficient big value of t_g based on condition (23). Another condition for string stability is $h(t) > 0$. With the chosen values of the parameter according to Table 1, the impulse response of the transfer function (22) is always positive (see Fig. 5). However, there is no direct design procedure that ensures that the impulse response $h(t)$ is non-negative. Only some indirect design tips are given in [11]. Two necessary conditions that must be satisfied by the transfer function $H(s)$ in order for impulse response to be non-negative are:

1. the dominant poles of the system should not be a complex conjugate pair;
2. there should not be any zeros of the system that are completely to the right of all poles of the closed-loop system.

B. *Proposed spacing policy*

The proposed spacing policy (PSP) considers the information of the controlled vehicle's state (inter-vehicle constant distance L, velocity v, deceleration d (always negative value), time delay of brake system t_b [15]) and safety coefficient k ($0.6 \le k \le 0.9$), which is relevant to road conditions. The value of k should be bigger if the road surface is wet or snowy to maintain safety spacing from preceding vehicle:

$$S_i = L + \frac{t_b}{1-k} v_i - \frac{k}{2d_i} v_i^2 = L + T_b v_i - \frac{k}{2d_i} v_i^2 \,. \tag{24}$$

The spacing error for the i-th vehicle under PSP is given by:

$$\delta_i = \varepsilon_i + L + T_b v_i - \frac{k}{2d_i} v_i^2 , \tag{25}$$

where $\varepsilon_i = x_i - x_{i-1}$. In order to ensure that the error δ_i converges to zero, the dynamics of δ_i is set as $\dot{\delta}_i = -\lambda \delta_i$, where λ is a positive control gain. Differentiating (25), the desired acceleration can be obtained as:

$$a_{i\_des} = -\frac{\dot{\varepsilon}_i + \lambda \delta_i}{T_b - \frac{k}{d_i} v_i} = -\frac{\dot{\varepsilon}_i + \lambda \delta_i}{T} . \tag{26}$$

TABLE I. PARAMETERS USED IN SIMULATION

Description	Label	Value
vehicle length	l	4.5 (m)
inter-vehicle constant distance	L	7 (m)
time-gap	t_g	2 (s)
control gain	λ	0.5
time lag of lower-level control system	τ	0.5 (s)
max. deceleration (braking capability)	d	-7 (m/s^2)
safety coefficient	k	0.7
time delay of brake system	t_b	0.15 (s)

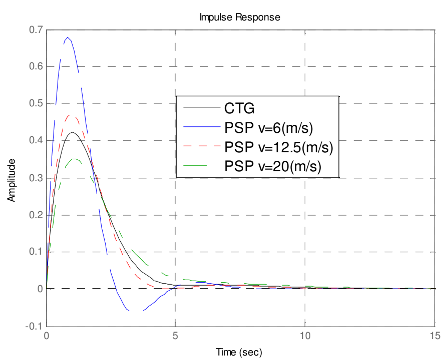

Figure 5. Impulse responses of spacing policies

In order to examine the string stability characteristic, PSP system (24) is linearized around the nominal vehicle velocity v_0, i.e. $v_{0,i} = v_{0,i+1} = v_0$ and the corresponding spacing is $S_{0,i} = S_{0,i+1} = S_0$ with $S_0 = L + T_b v_0 - \frac{k}{2d_i} v_0^2$. Assume that at a certain time instant the velocity of the preceding vehicle is perturbed. This perturbation propagates upstream to the tail of the platoon. Let $v_i = v_0 + \Delta v_i$, $\dot{v}_i = \Delta \dot{v}_i$ and $S_i = S_0 + \Delta S_i$, $\dot{S}_i = \Delta \dot{S}_i$, then for linearized system the transfer function of spacing error is the same as that of the velocity variation [14]:

$$H(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{\Delta v_i(s)}{\Delta v_{i-1}(s)} . \tag{27}$$

Under the assumptions that deviations from nominal spacing and vehicle velocity are small, substituting (26) into (18) and differentiating obtained equation we get:

$$\tau \ddot{a}_i + \dot{a}_i = -\frac{\ddot{\varepsilon}_i + \lambda \dot{\delta}_i}{T} . \tag{28}$$

Using the relations $v_i = v_0 + \Delta v_i$, $\dot{v}_i = \Delta \dot{v}_i$ and taking Laplace transforms of the above equation, we get transfer function of the velocity variation:

$$H(s) = \frac{\Delta v_i(s)}{\Delta v_{i-1}(s)} = \frac{s + \lambda}{T\tau s^3 + Ts^2 + (1 + \lambda T)s + \lambda} . \tag{29}$$

In order to ensure the string stability, the conditions (6) and (7) should be satisfied. The magnitude of the transfer function (26) will be no bigger than 1 only if:

$$T \ge 2\tau \quad \Rightarrow \quad \frac{t_b}{1-k} - \frac{k}{d_i} v_i \ge 2\tau . \tag{30}$$

For the chosen values of the parameter according to Table 1, the condition (30) is satisfied if $v_i \ge 5$ (m/s). Another condition for string stability is that the impulse response of the transfer function (29) is $h(t) > 0$. The impulse responses of (29) at different velocities are shown in Fig 5. The condition $h(t) > 0$ is satisfied when velocity is higher than 12.5 (m/s). The final string stability condition is $v_i \ge 12.5$ (m/s).

We assume that inter-vehicle spacing, relative velocity between two vehicles, velocity of the controlled vehicle (all those measurements can provide on board radar and sensors) and braking capability of controlled vehicle (can be obtained from the vehicle manufactures or by experiments) are available for upper controller.

V. INFLUENCE OF SPACING POLICIES ON TRAFFIC FLOW

A. *Traffic flow stability of constant time-gap spacing policy*

For the CTG spacing policy, the steady traffic density is given by

$$\rho = \frac{1}{L + t_g v} . \tag{31}$$

By solving v in terms of ρ from (31) and multiplying obtained vehicle velocity v with traffic density ρ the following equation of the traffic flow is obtained:

$$Q = \rho v = \frac{1 - \rho L}{t_g} . \tag{32}$$

The condition $\partial Q / \partial \rho > 0$ should be satisfied for stable traffic flow. In (32) the variable $\partial Q / \partial \rho = -L / t_g$ always has negative value (see Fig. 6), which means the CTG spacing policy is always traffic flow unstable.

B. *Traffic flow stability of the proposed spacing policy*

The traffic density at the steady state for PSP is given by

$$\rho = \frac{1}{L + T_b v - \frac{k}{2d} v^2}. \tag{33}$$

Two solutions are obtained solving (33) for v in terms of ρ. Only positive solution is acceptable:

$$v = \frac{dT_b}{k} - \frac{d}{k}\sqrt{T_b^2 + \frac{2k}{d}\left(L - \frac{1}{\rho}\right)}. \tag{34}$$

Then we obtain the traffic flow

$$Q = \rho v = \frac{\rho d T_b}{k} - \frac{\rho d}{k}\sqrt{T_b^2 + \frac{2k}{d}\left(L - \frac{1}{\rho}\right)}. \tag{35}$$

PSP holds a stable traffic flow ($\partial Q / \partial \rho > 0$) when the traffic density ρ is below a critical density $\rho_{cr} \approx 0.065$ (vehicle/m) (see Fig. 6).

In Fig. 7, the traffic flow Q is plotted in dependency of the traffic velocity v for both spacing policies. It can be seen that by lowering the traffic velocity, the traffic flow capacity grows with the proposed spacing policy (in the stable region – above $v_i \geq 12.5$ (m/s)) and decreases with the CTG spacing policy. For example, at the traffic velocity of 22,2 (m/s), i.e. 80 (km/h), the PSP ensures about 20% higher capacity of the traffic flow than the CTG spacing policy.

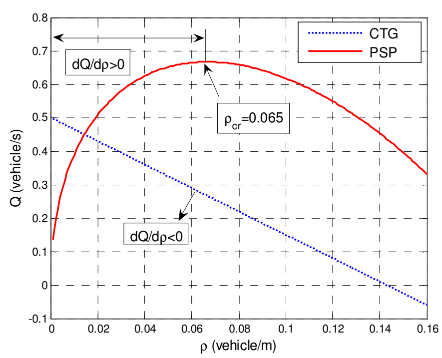

Figure 6. $Q - \rho$ curves of PSP and CTG spacing policy

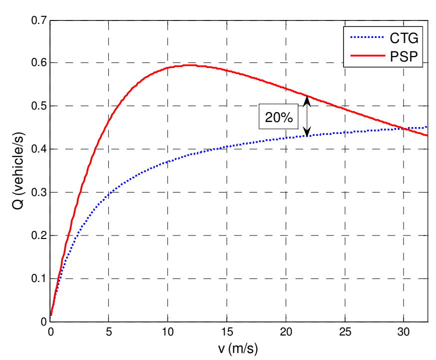

Figure 7. $Q - v$ curves of PSP and CTG spacing policy

VI. Conclusion

A nonlinear longitudinal spacing control policy is proposed that ensures string stability and traffic stability of vehicles in a platoon. It explicitly takes into account the vehicle's braking capability and road conditions and could be used for a wide range of vehicle's velocities. Therefore, it is suitable both for highway and urban traffics. Comparing to the standard constant time-gap spacing policy, the proposed spacing policy ensures increased traffic capacity and stable traffic flow.

The safety coefficient is bounded on $0.6 \leq k \leq 0.9$ because: (i) if $k = 0$, proposed spacing policy (24) degenerates into a standard constant time-gap spacing policy. Nevertheless, the time-gap would then be very short and the string stability (according to (27)) and the traffic flow stability would no longer be satisfied. (ii) if $k = 1$, desired inter-vehicle distance tends to infinity (because of the second term in (24)).

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